Constitutive and Damage Accumulation Modeling

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Acknowledgement

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Sponsor: **Semiconductor Research Corporation**
Advance Micro Devices, Inc.

Outline

- Objective
- Constitutive and damage model
- Preliminary application results
- Data needed for lead free solders

Objectives

- Build constitutive model with damage so that numerical simulation of conditions encountered in use becomes possible (since experimental simulation is not a choice in some circumstances)
- Develop materials science based damage formation and evolution law to overcome size limitation
- Develop theory that gives realistic life prediction and metallurgical directions

Inelastic Strain

Due to presence of both creep and plasticity in solders, Unified Creep and Plasticity strain rate is used:

$$\dot{\boldsymbol{\varepsilon}}^{in} = \sqrt{\frac{3}{2}} A \left(\frac{S_V}{d} \right)^n e^{B \left(\frac{S_V}{d} \right)^{n+1}} \Theta(T) \mathbf{N} \qquad \Theta = e^{-\frac{Q}{RT}} \text{ for } T \geq \frac{T_m}{2}$$

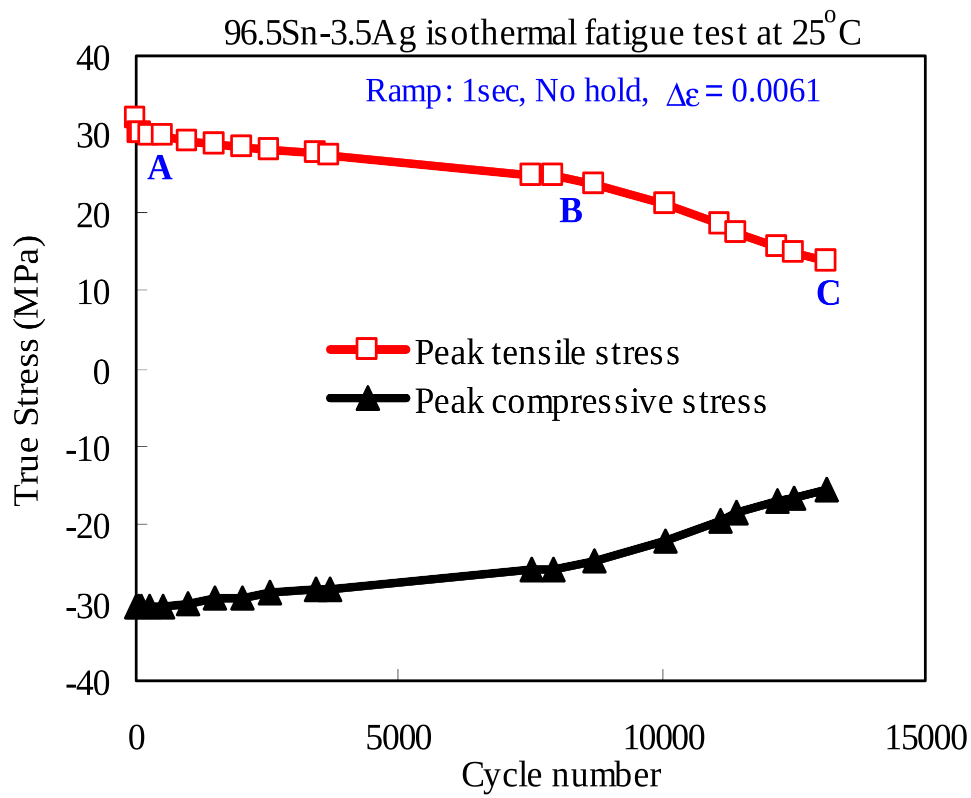
$$S_v = \left\langle \sqrt{\frac{3}{2}} \left\| \mathbf{S} - \boldsymbol{\alpha} \right\| - \underline{R} \right\rangle \qquad \text{with} \qquad \dot{\boldsymbol{\alpha}} = \mu \left\| \dot{\boldsymbol{\varepsilon}}^{in} \right\| \mathbf{N} - \beta \boldsymbol{\alpha}$$

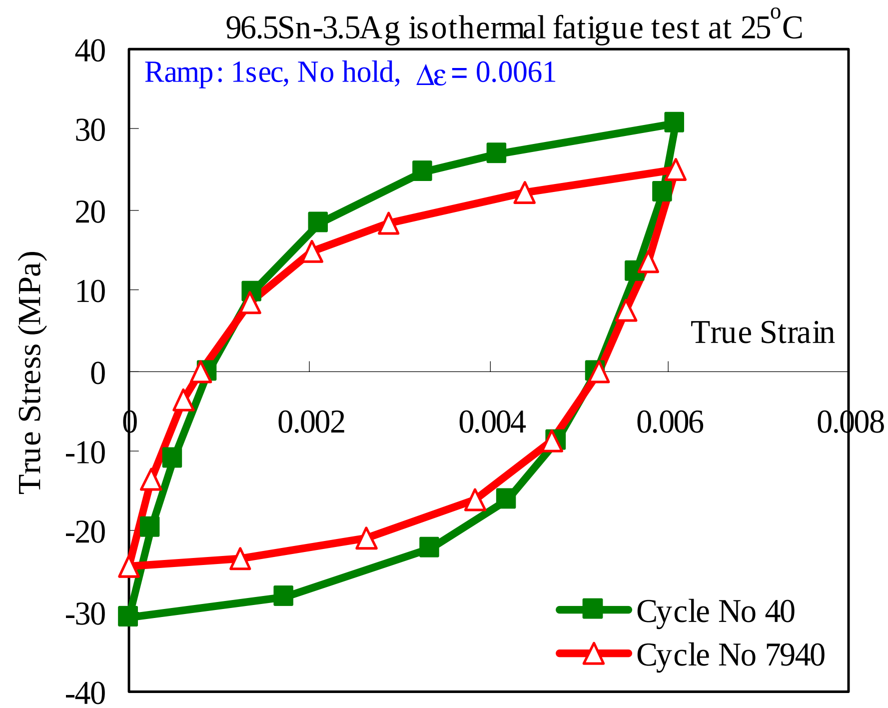
$$\mathbf{N} = \frac{\mathbf{S} - \boldsymbol{\alpha}}{\left\| \mathbf{S} - \boldsymbol{\alpha} \right\|} \qquad \text{Hooke's Law: } \dot{\boldsymbol{\sigma}} = \mathbf{C} : \left(\dot{\boldsymbol{\varepsilon}}^{tot} - \dot{\boldsymbol{\varepsilon}}^{in} - \alpha^T \dot{T} \mathbf{I} \right)$$

where **S** is the deviatoric stress tensor, and Θ is a diffusivity term, α is back-stress, $\underline{R}$ is the radius of the yield surface.

Mechanical Performance: 96.5Sn-3.5Ag

Damage along with cyclic loading shown in UNIAXIAL test

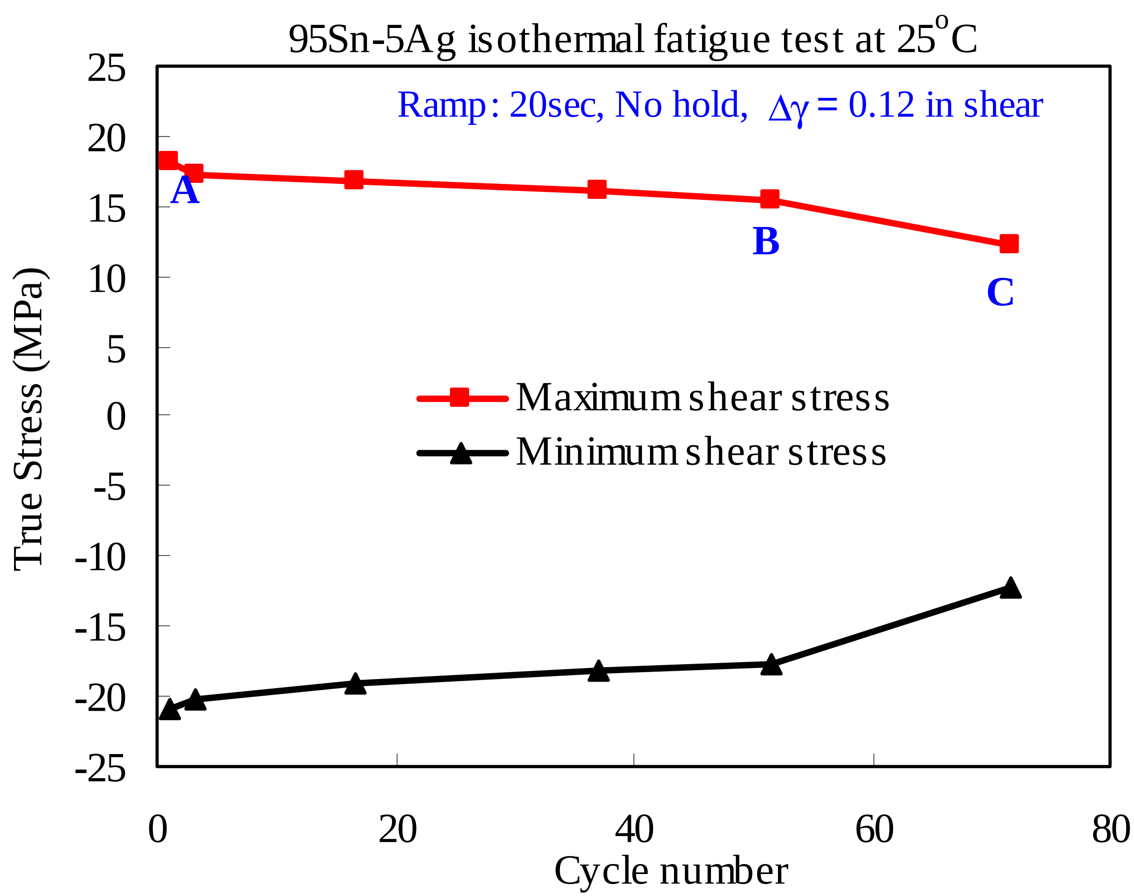




Hysteresis loops at A and B

(Data from H. Mavoori)

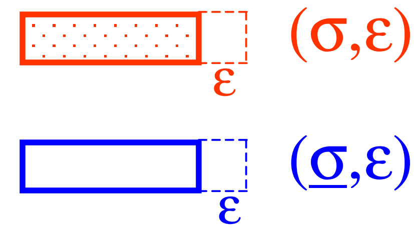
Mechanical Performance: 95Sn-5Ag



Damage along with cyclic loading shown in SHEAR strain-controlled test (**Data from J. Liang, et al,** *Fatigue Fract. Engng. Mater. Struct.*, **19:1401 (1991)**)

Damage mechanics & D(ω)

Strain equivalence is used, so that:



$$\underline{\boldsymbol{\sigma}} = \mathbf{M}^{-1} : \boldsymbol{\sigma} \quad \text{and} \quad \boldsymbol{\varepsilon} = \mathcal{G}(\underline{\boldsymbol{\sigma}})$$

For isotropic damage: $\mathbf{M} = (1\text{-}D)\mathbf{II} \longrightarrow \underline{\boldsymbol{\sigma}} = \frac{1}{1-D}\boldsymbol{\sigma}$

Corresponding Changes:

$$S_v = \left\langle \sqrt{\frac{3}{2}} \frac{1}{1-D} \|\mathbf{S} - \boldsymbol{\alpha}\| - \underline{R} \right\rangle \qquad \dot{\alpha} = (1-D)\mu \|\dot{\boldsymbol{\varepsilon}}^{in}\| \mathbf{N} - \beta \boldsymbol{\alpha}$$

$$\dot{\boldsymbol{\sigma}} = (1-D)[\mathbf{C}(\theta) : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{in} - \chi \dot{\theta} \mathbf{I}) + \mathbf{C}^{-1} : \boldsymbol{\sigma} : \frac{\partial \mathbf{C}}{\partial \theta} \dot{\theta}]$$

Micro- and Macro-cracks

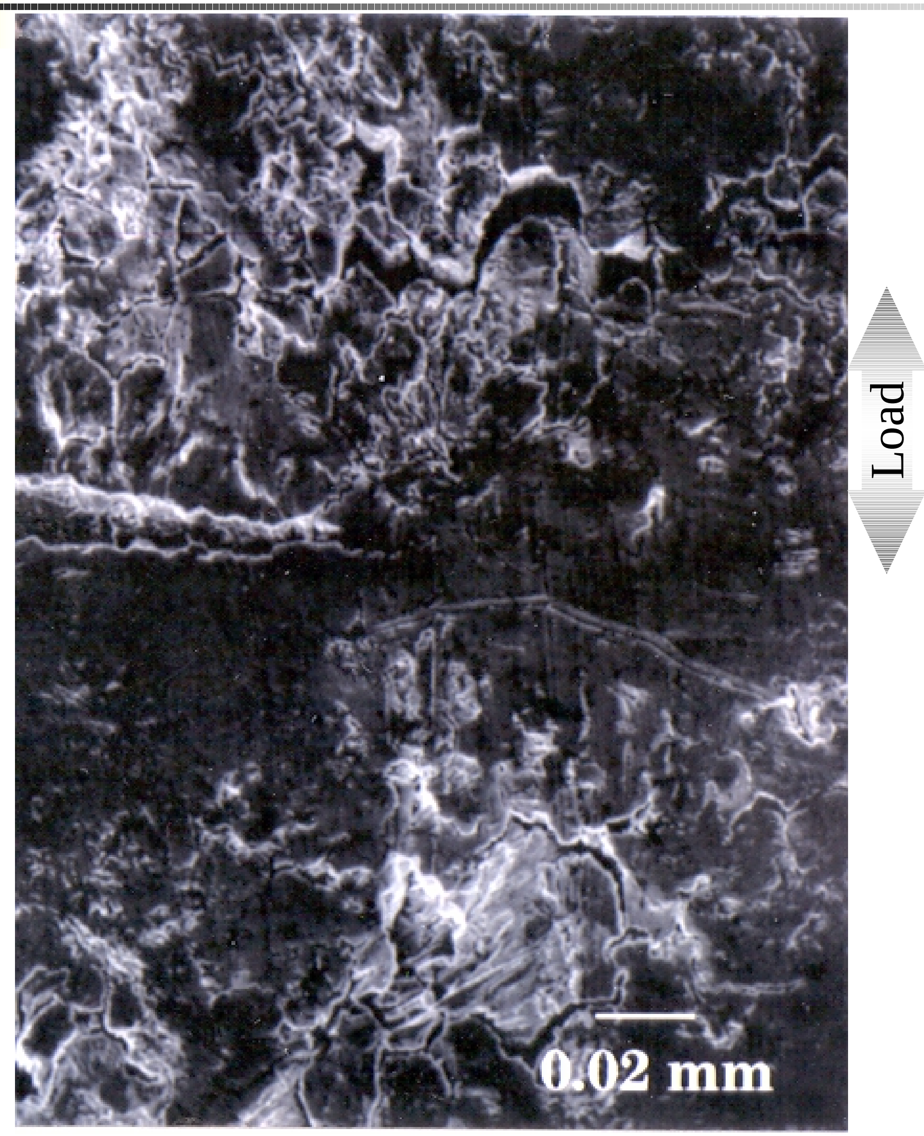


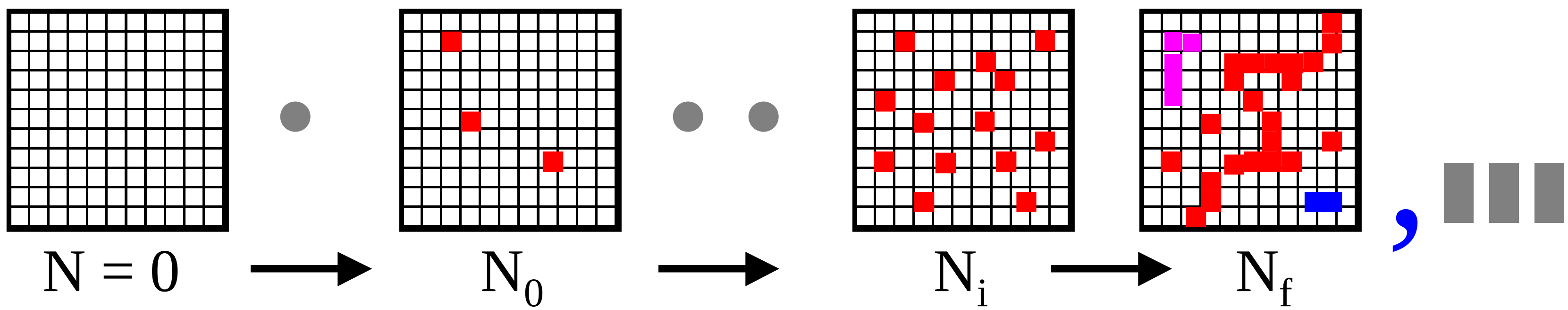
Photo from H. Mavoori Ph.D. Dissertation (NU1996), Fig.6.3

Microcracks and macrocracks (96.5Sn-3.5Ag at $\Delta\varepsilon$=0.003, ramp:1sec, no hold, N/N_f=1.06, at 25°C)

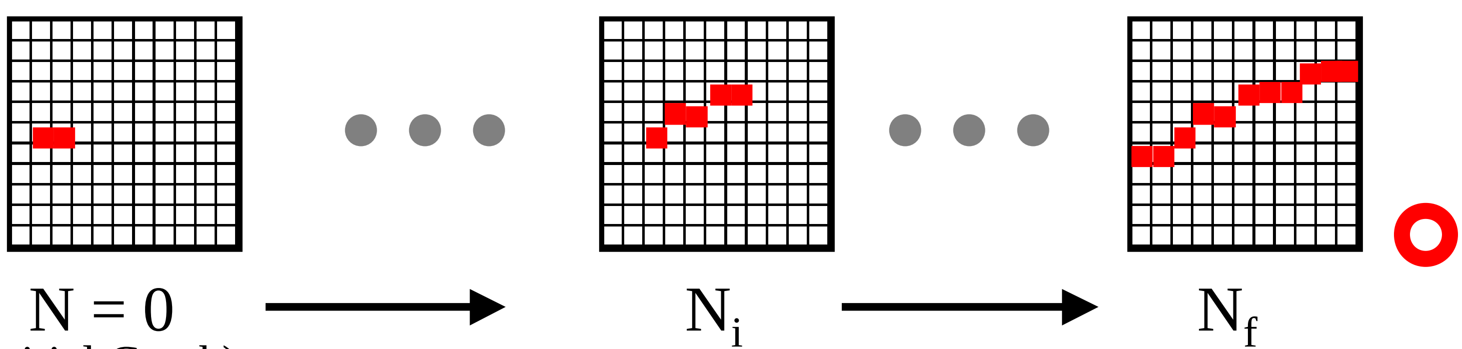
At the end of the test, specimen is full of microcracks. Some of the microcracks have coalesced into macrocracks.

Damage Model: Percolation

Damage accumulation: Percolation or a serial of independent local events



Fracture mechanics approach



Nucleation-Extension

Mura and Nakasone: $\Delta G = -W_1 - W_2 + 2\gamma A$

(J. Appl. Mech., 57:1(1990))

$$\frac{\partial(\Delta G)}{\partial n} = 0$$

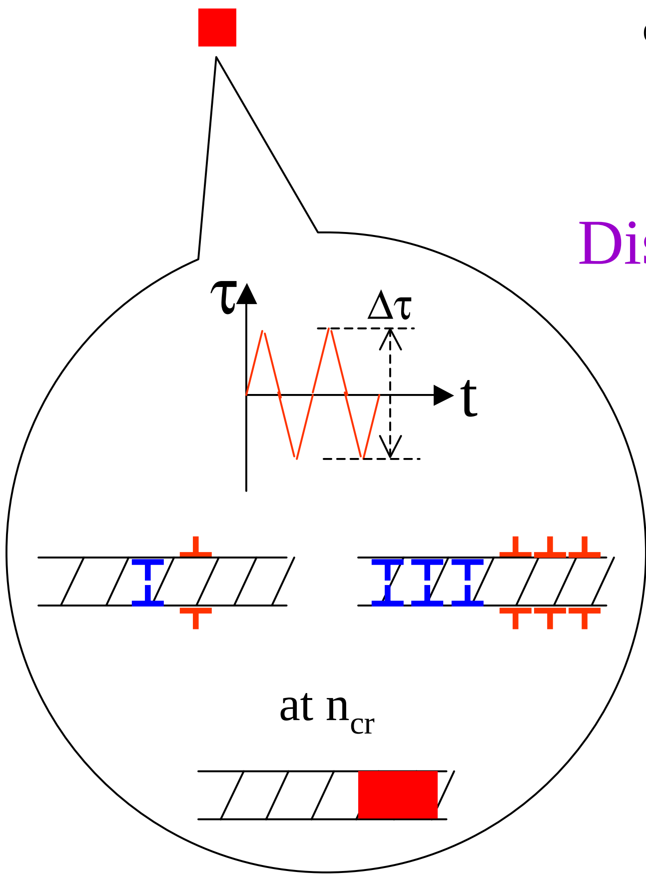
Dislocation Energy Density – Fine (Fine: Scripta mater. 42:1007 (2000))

$$W_2 = 2\gamma_d A$$

Cracks formation cycle n_{cr}(Assume penny-shape shear cracks dominate):

$$W_1 = \frac{64(1-\nu^2)\Delta\tau^2 a^3}{3(2-\nu)E}$$

$$n_{cr} = \frac{(2-\nu)\pi(\gamma-\gamma_d)E^2}{32(1-\nu^2)^2\bar{d}(\Delta\tau - 2\tau_f)\Delta\tau^2}$$

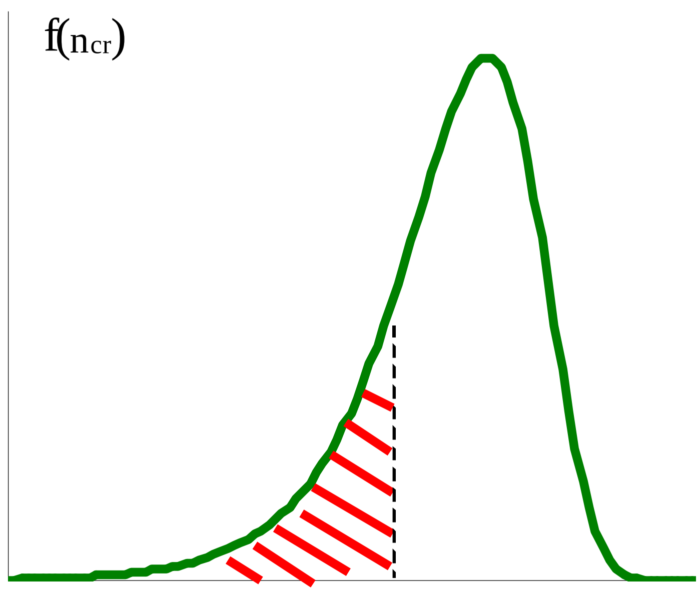


Dislocations of vacancy type form a crack

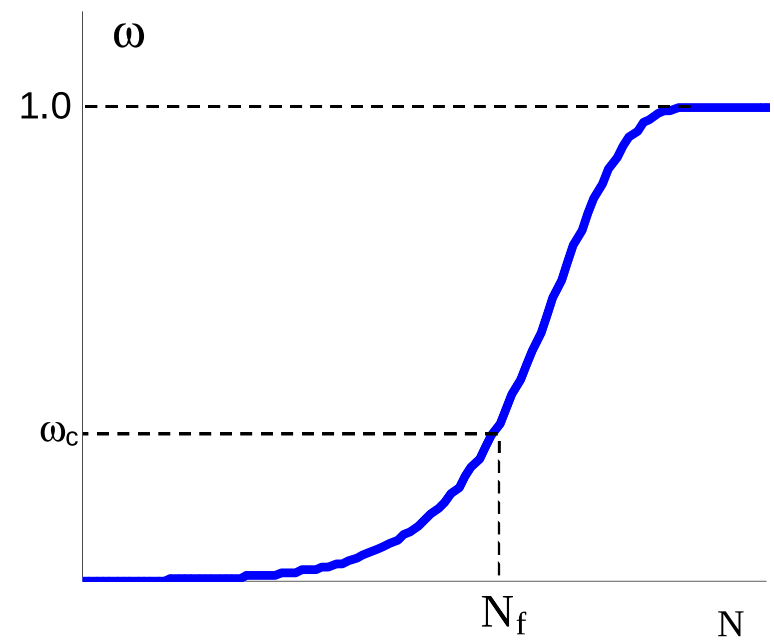
Physical damage metric

$$\omega = \int_0^N f(n_{cr})\,dn_{cr}$$

$f(n_{cr})$ is a probability distribution function for n_{cr}, or the probability of a grain or cell that has a crack formation cycle number of n_{cr}.



Critical cycle distribution



Physical damage metric

Calculation of D from ω

If $D = g(\omega)$, then $D\big|_{\omega=0} = g(0) = 0$

Thus: $D = g'(\omega)\omega + \frac{1}{2} g''(\omega)\omega^2 + \cdots$

A special power law form: $D(\omega) = D_c \left(\frac{\omega}{\omega_c} \right)^{\eta}$

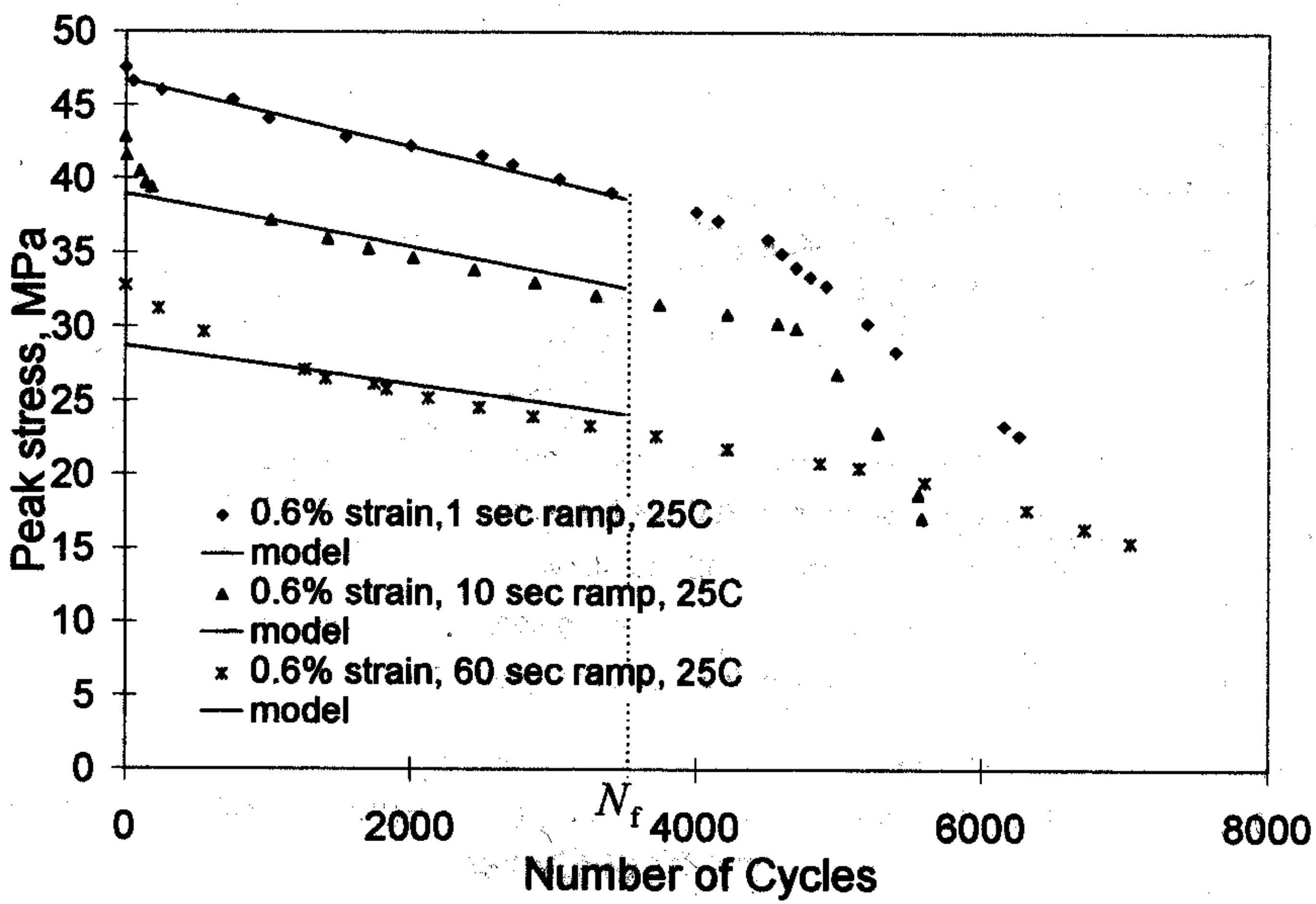
For bulk specimen, $f(n_{cr})$ is a constant. Let $f(n_{cr}) = C$, then

$$\omega = \int_0^N f(n_{cr}) dn_{cr} = \int_0^N C dn_{cr} = CN \quad \rightarrow \quad \omega / \omega_c = N / N_f$$

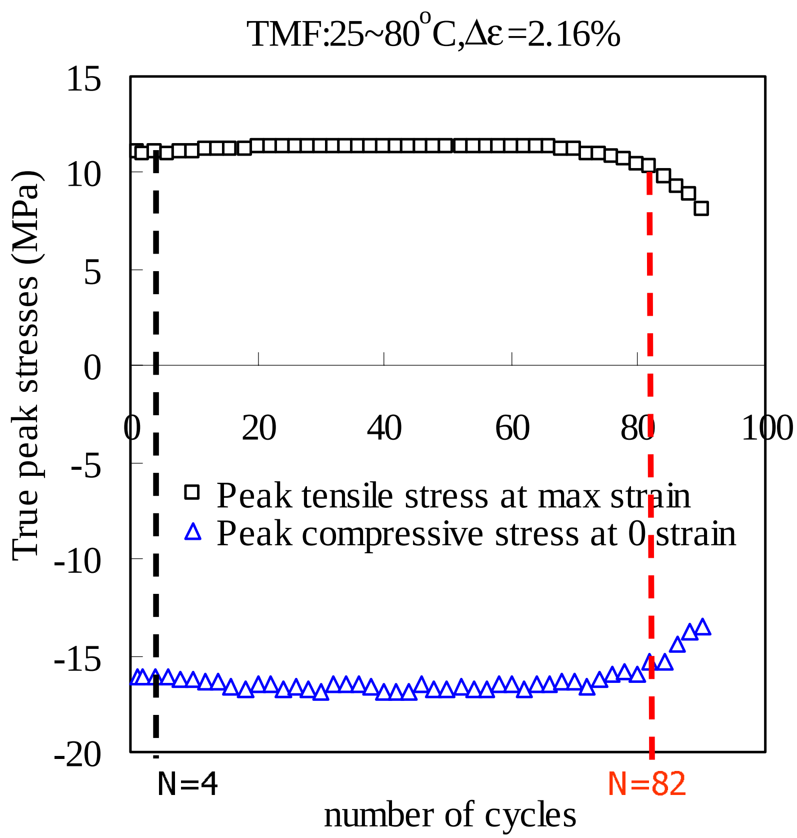
$$\rightarrow \quad D = D_c \left(\frac{N}{N_f} \right)^{\eta}$$

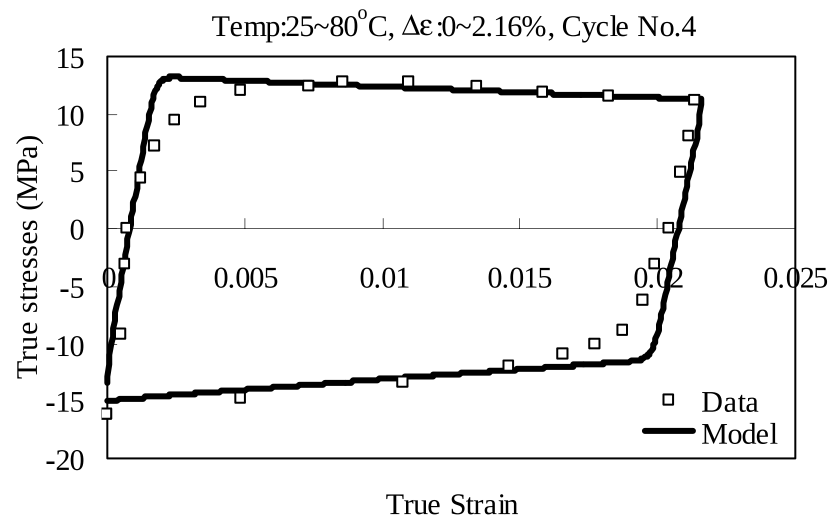
63Sn-37Pb Eutectic Solder

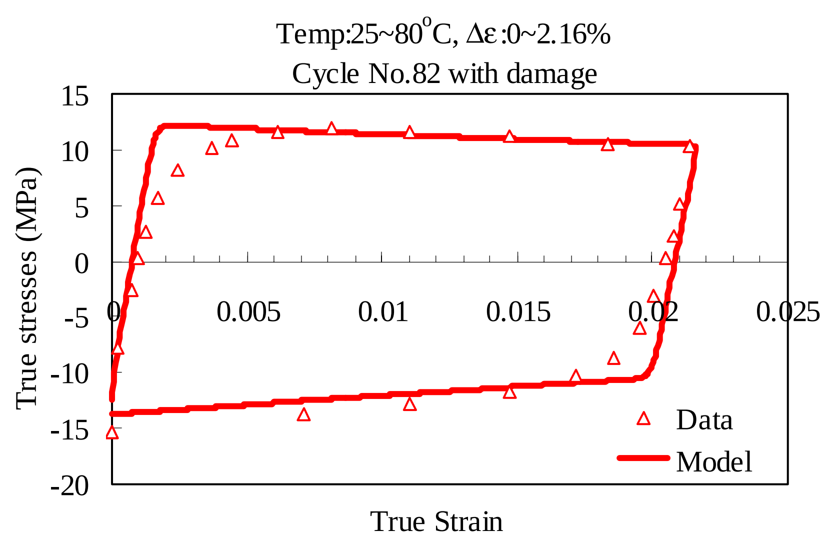
V. Stolkarts et al. / J. Mech. Phys. Solids 47 (1999) 2451–2468



96.5Pb-3.5Sn high-lead solder

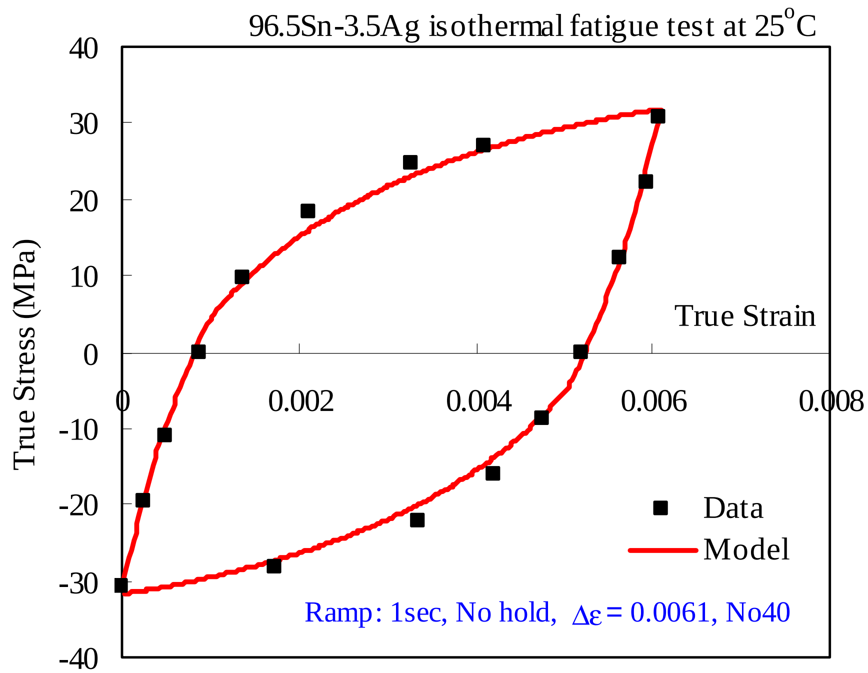




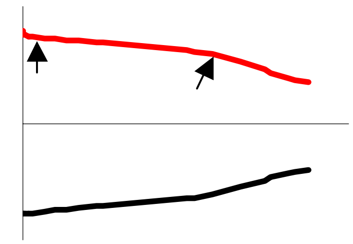


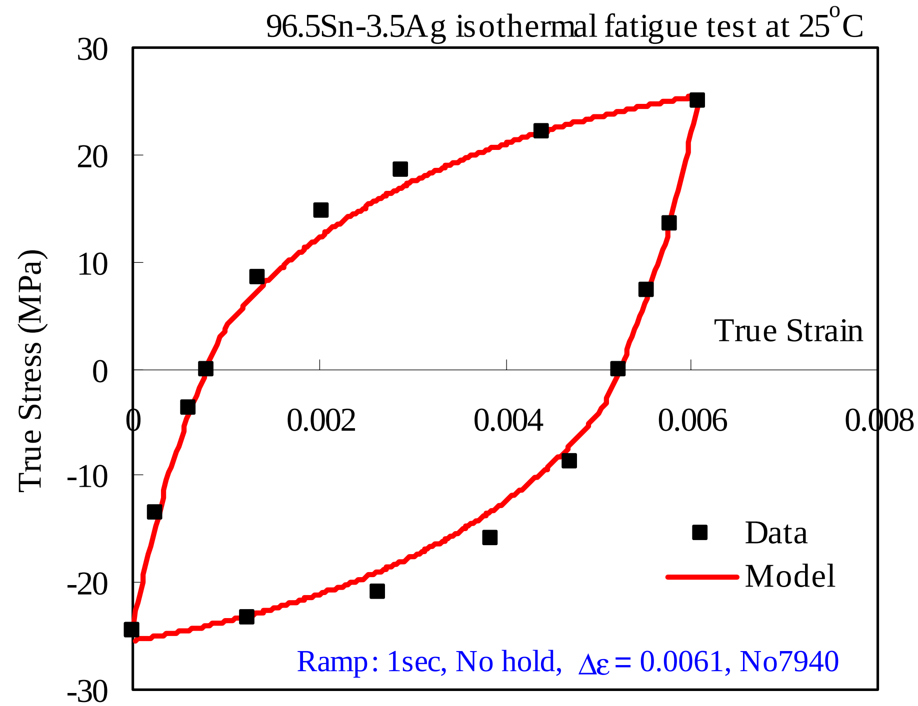
TMF data from Lawrence R. Lawson

96.5Sn-3.5Ag lead-free solder



Cycle No 40, no damage





Cycle No 7940, damage

Isotermal data from Hareesh Mavoori

Database and Model

Mechanical properties:
Young's Moduli
Tensile and shear strengths
Elongations to failure
Stress-relaxation and Creep
Activation energy
Thermal coefficient
Stress exponents
Creep exponents
Other material constants

Fatigue:
Isothermal fatigue data
Thermo-mechanical fatigue

Fractography and
Microstructure data

× **strain rate**
temperature

Data base ⟷ **Model** ⟷ **Product design**

Current state

- Extensive databases exist for Pb-based solders, but almost none are available for lead-free solder (CINDAS collected some data but not enough)
- Tests not designed for investigation of damage formation and evolution
- Current modeling is without damage or with damage assumed
- Fatigue characterization is largely empirical and not materials science based

Mechanical Properties

Properties: Young's Moduli, Shear moduli, Ultimate properties (such as yield, tensile and shear strengths, elongation to failure, etc.), Stress-relaxation and creep, stress and creep exponents, thermal expansion and capacity constants, and others.

Current state:

- Data are available but not sufficient
- Specimens, strain rates, temperatures, set-ups and procedures differ
- Modeling, validation and comparison not easy to determine
- Good correlation between bulk and joints (H. Solomon from GE and R. Gagliano, et al, from Northwestern)

What are needed:

- Creep and relaxation and stress-strain curves under uniaxial and shear
- Rate and temperature dependence
- Standardized experimental procedures.
- Data on actual interconnects (quantitative data in shear)
- Size effects may be needed from actual joints

Fatigue tests

Tests:

- Isothermal
- Thermomechanical
- Shear
- Uniaxial (mostly)
- Ramp time, hold time
- Temperature range

Current state:

- Different failure definitions, test procedures (most uniaxial)
- Phenomenological formula must be proved effective for smaller joints and/or under multi-axial stress conditions
- Intermediate damaged state is seldom investigated

What are needed:

- Standardize definition of fatigue failure for data sharing
- Intermediate state investigation to monitor damage evolution
- Design various microstructure specimens from damage perspective to seek better metallurgical measures
- Test conditions to duplicate field operating conditions

Fractography

- Discover microcrack origin, nucleation, evolution, coalescence phenomena
- Relate the mechanical performance and the damage state and its evolution
- Understand the failure mechanism

What is needed: Testing oriented to damage mechanism identification.

Microstructure

Microstructure is very important to understand and model the failure mechanisms and can lead to better design:

$$n_{cr} = \frac{(2-\nu)\pi(\gamma-\gamma_d)E^2}{32(1-\nu^2)^2\bar{d}(\Delta\tau-2\tau_f)\Delta\tau^2}$$

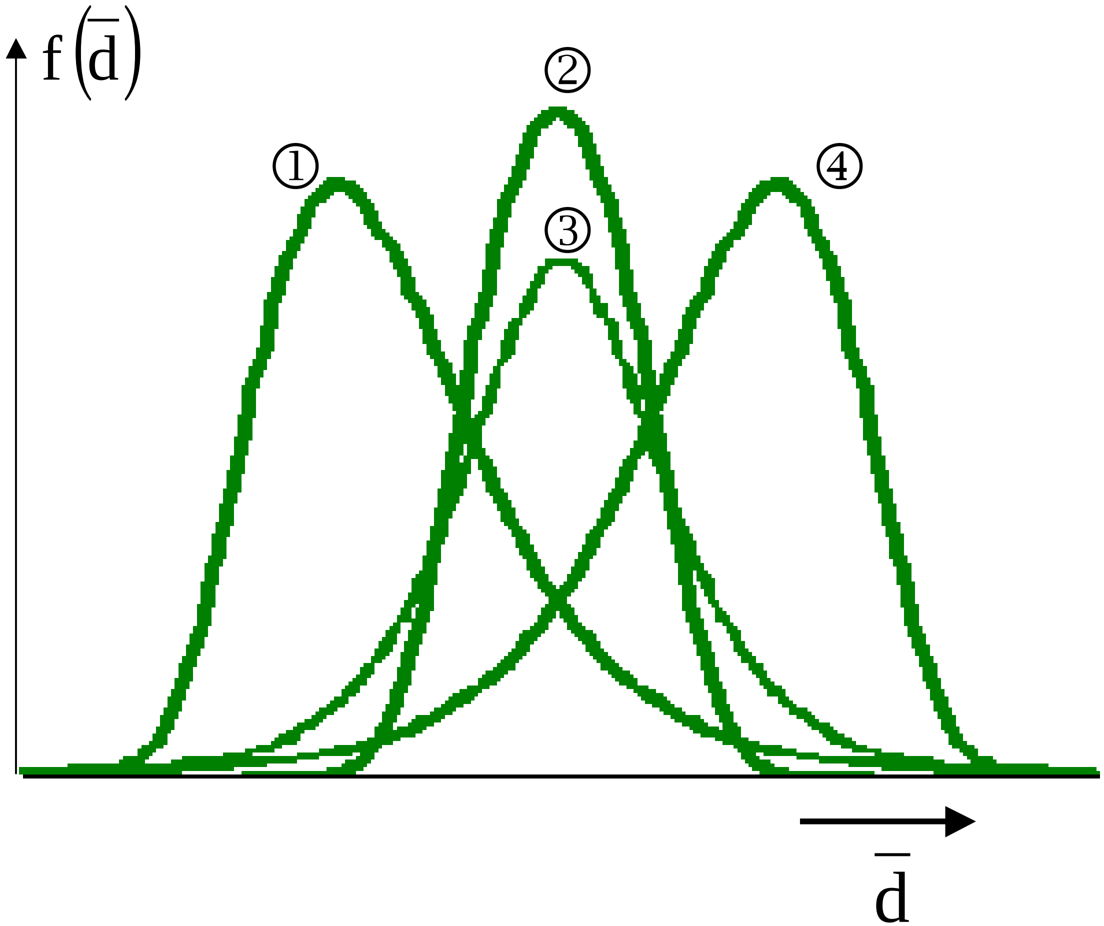
- Grain size and its distribution
- Orientation distribution
- Microstructure evolution

$\bar{d}\downarrow \Rightarrow n_{cr}\uparrow$

$\Delta\tau\downarrow \Rightarrow n_{cr}\uparrow$

What are needed: Tests on various but well-characterized microstructures

Grain size and distribution



① **excellent**

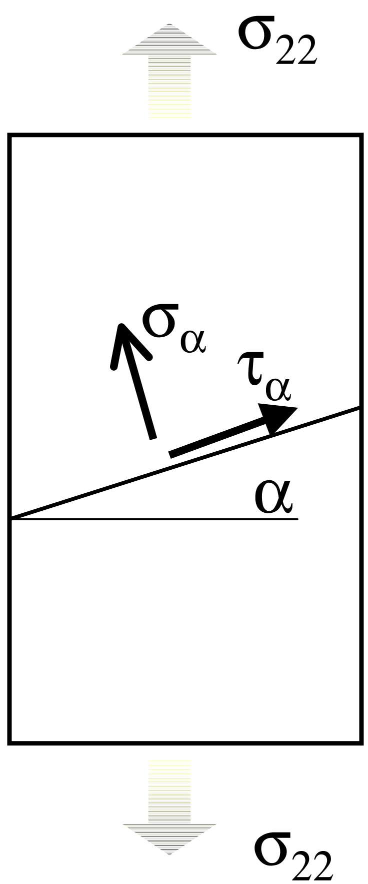
② **good**

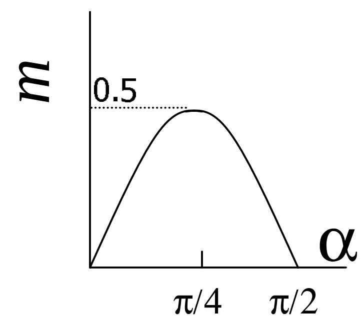
③ **fair**

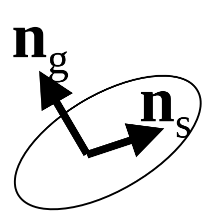
④ **poor**

Use metallography to find out $f(\bar{d})$

Grain orientation







Local shear stress range depends on the grain's slip plane orientation:

$$\Delta\tau_\alpha = 0.5 \sin 2\alpha \Delta\sigma_{22}$$

Generally:

$$\Delta\tau = \mathbf{n}_s \cdot \Delta\sigma \cdot \mathbf{n}_g$$

$\mathbf{n}_g$ is the normal to the glide plane while $\mathbf{n}_s$ is the slip direction.

Remark: Stereology, numerical simulation, and others, to find grain orientation distribution function

Metallurgical aspects

- From damage accumulation perspective, metallurgical measures, such as reduced grain size, can improve fatigue life of solder structures.
- Knowledge from lifetime improvement by metallurgical measures can be used to improve the accuracy of the damage accumulation method.
- Collaboration between disciplines of materials science and mechanics are needed.

How accurate can it be?

With the data input from prior discussion, a damage accumulation model can be constructed.

A question is: Can this model make accurate predictions of reliability?

Answer:

The model may give statistically significant predictions.

Conclusion

- A database for lead-free solders in needed
- Testing oriented to damage mechanism should be conducted
- Numerical simulation and computational analysis should play a more important role in the modeling when direct measurement is not available